For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Find the complement of the supplement of $\frac{7\pi}{9}$ radians, in radians.

   (A) $\frac{2\pi}{9}$ (B) $\frac{5\pi}{18}$ (C) $\frac{\pi}{6}$ (D) $\frac{7\pi}{18}$ (E) NOTA

2. Find the length of the diagonal in a rectangular prism with length 12 units, width 21 units, and height 28 units.

   (A) $3\sqrt{65}$ (B) 39 (C) 37 (D) $\sqrt{1379}$ (E) NOTA

3. Find the volume of an octagonal pyramid with height 4 and regular octagonal base with side length 4.

   (A) $\frac{128+128\sqrt{2}}{3}$ (B) $\frac{64+64\sqrt{2}}{3}$ (C) $32+32\sqrt{2}$ (D) $128+128\sqrt{2}$ (E) NOTA

4. Find the area of the quadrilateral defined by the points $(4, 19)$, $(7, 31)$, $(9, 39)$, and $(-6, -21)$.

   (A) 45 (B) 30 (C) 20 (D) 15 (E) NOTA

5. Let ΔABC have side lengths $\overline{AB} = 20$, $\overline{BC} = 18$, and $\overline{AC} = 16$. Let point D be on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Find the length of segment $\overline{BD}$.

   (A) 8 (B) 10 (C) 6 (D) 12 (E) NOTA

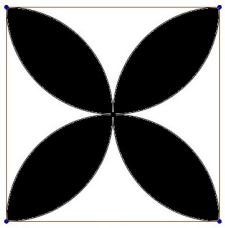
6. Find the sum of the exterior angles in the first convex polygon found in Euclid's *Elements*.

   (A) $90°$ (B) $360°$ (C) $450°$ (D) $540°$ (E) NOTA

7. William's house is in the shape of a 15 by 12 centimeter rectangle. He ties his pet sheep to one corner of the house with a 20 centimeter long rope outside the house. What is the total area, in square mm, within which his sheep can roam, given that it cannot pass through the walls of his house?

   (A) 128900π (B) 32225π (C) $\frac{1289\pi}{4}$ (D) $\frac{689\pi}{2}$ (E) NOTA

8. Joshua really likes flowers. Given this diagram, where 4 semicircles of diameter equal to the side length of the square are inscribed in a square, find the area of the shaded region if the square has side length 8.



   (A) $128-32\pi$ (B) $96-16\pi$ (C) $16\pi-32$ (D) $32\pi-64$ (E) NOTA

9. Granny has a pile of apples, stacked in layers. The top layer has one apple, the second layer has four apples, the third layer has nine apples, and so on. If each apple is perfectly spherical in shape and has radius 1 inch, and the pile has 20 layers, find the total volume of apples in the pile.

   (A) $\frac{11480\pi}{3}$ (B) $\frac{9880\pi}{3}$ (C) 2812π (D) $\frac{2870\pi}{3}$ (E) NOTA

10. Given the triangle ΔABC defined by $\overline{AB} = 8$, $\overline{AC} = 12$, and $\angle ACB = 30°$, find the positive difference between the maximum and minimum areas that the triangle can define.

(A) 12 (B) $2\sqrt{7}$ (C) $\frac{768}{25}$ (D) $\frac{1536}{25}$ (E) NOTA

11. Sam has a right cone-shaped container with height 12 units and base radius 4 units. The cone is upright (the base is on the floor) and can be filled with water from its tip. The water trickles in at a non-constant rate such that the height of the water in the cone increases at a rate of 0.2 units per second. After 30 seconds, find the volume of water in the cone.

(A) 28π (B) 52π (C) 56π (D) 64π (E) NOTA

12. Nihar's birthday is the 14th, so he is having a birthday party! He has a cake in the shape of a cylinder, and wants to cut it using 4 cuts. What is the maximum number of pieces into which he can cut the cake?

(A) 9 (B) 12 (C) 15 (D) 18 (E) NOTA

13. Circles of radius 5, 5, and 8 are mutually externally tangent. A fourth circle with radius r is externally tangent to all three previous circles. Find r.

(A) $\frac{28}{9}$ (B) $\frac{8}{9}$ (C) $\frac{13}{9}$ (D) $\frac{4}{3}$ (E) NOTA

14. Find the largest positive integer divisible by all positive integers less than its square root.

(A) 1 (B) 15 (C) 27 (D) 36 (E) NOTA

15. An *emirp* is a prime number that results in a different prime number when its digits are reversed (and, if you notice, is *prime* spelled backwards). Palindromic primes and single digit primes are not counted as emirps. Find the fifth smallest emirp; in other words, in a sequence of emirps in increasing order, find the fifth emirp in the sequence.

(A) 31 (B) 37 (C) 71 (D) 73 (E) NOTA

16. Evaluate the following infinite arithmetico-geometric series: $\frac{1}{2} + \frac{3}{8} + \frac{5}{32} + \ldots$.

(A) 2 (B) $\frac{4}{9}$ (C) $\frac{10}{9}$ (D) $\frac{2}{3}$ (E) NOTA

17. What is the 5th term in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^7$?

(A) $\frac{35}{x^6}$ (B) $\frac{21}{x^{11}}$ (C) $\frac{21}{x^6}$ (D) $\frac{35}{x^{11}}$ (E) NOTA

18. What is the sum of the cubes of the roots of $6x^5 - 3x^4 + 2x^3 + 4x^2 - 8x + 10 = 0$?

(A) $\frac{11}{12}$ (B) $-\frac{41}{24}$ (C) $-\frac{3}{8}$ (D) $\frac{1}{2}$ (E) NOTA

19. Evaluate:

$$\begin{vmatrix} 3 & 4 & -2 \\ 6 & 11 & 2 \\ 1 & 3 & 5 \end{vmatrix}$$

(A) 49 (B) 245 (C) 21 (D) 56 (E) NOTA

20. Find x if $\log_4(\log_2 x) = \log_{\frac{1}{4}}(\log_3 2)$.

(A) 4 (B) $\frac{4}{9}$ (C) $\frac{1}{3}$ (D) 3 (E) NOTA

21. Solve for x if $\sqrt[3]{4x+9} - \sqrt[3]{4x-9} = \sqrt[3]{9}$.

(A) $\pm\frac{\sqrt{2}}{2}$ (B) $\pm\frac{\sqrt{3}}{2}$ (C) $\pm\frac{\sqrt{15}}{2}$ (D) $\pm\frac{\sqrt{21}}{2}$ (E) NOTA

22. What is the sum of the values of x that satisfy $|3x-7| = |4x-5|$?

(A) $-\frac{2}{7}$ (B) $\frac{5}{7}$ (C) 1 (D) $\frac{1}{7}$ (E) NOTA

23. Find the equation of the line, in slope-intercept form, that is tangent to $(x-11)^2 + (y-6)^2 = 20$ at $(15, 4)$.

(A) $y = -\frac{1}{2}x + \frac{23}{2}$ (B) $y = 2x - 26$ (C) $y = \frac{1}{4}x + \frac{31}{4}$ (D) $y = 4x - 56$ (E) NOTA

24. What is the equation of the parabola with directrix $x = -3$ and focus $(5, 2)$?

(A) $y = \frac{3}{8}x^2 + \frac{1}{4}x - \frac{3}{4}$ (B) $y = \frac{1}{2}x^2 - \frac{5}{8}x + \frac{9}{16}$ (C) $x = \frac{1}{8}y^2 - \frac{3}{2}y + \frac{3}{16}$ (D) $x = \frac{1}{16}y^2 - \frac{1}{4}y + \frac{5}{4}$ (E) NOTA

25. Evaluate:

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 6n + 8}$$

(A) $\frac{7}{12}$ (B) 1 (C) $\frac{4}{15}$ (D) $\frac{5}{16}$ (E) NOTA

26. Find the magnitude of $16 + 63i$.

(A) 79 (B) $4 + 3\sqrt{7}$ (C) $\sqrt{79}$ (D) 4 (E) NOTA

27. Given a 5×12 grid made up of unit squares, how many paths can be taken from the bottom left corner to the top right corner if the grid can only be traversed by going a unit up or a unit right (if either is possible) at every point?

(A) 742560 (B) 30940 (C) 6188 (D) 1547 (E) NOTA

28. The graph of which of the following equations depicts a pair of intersecting lines?

(A) $3x^2 = 4y^2$ (B) $5x^2 + 2y^2 + 1 = 0$ (C) $x^2 = 1$ (D) $x^2 + 2xy + y^2 = 0$ (E) NOTA

29. What is the coefficient of the third term in the expansion of $(x+y)^{\frac{1}{3}}$?

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$ (E) NOTA

30. Consider $\triangle ABC$ with vertices $A(1,2)$, $B(4,7)$, and $C(6,2)$. If a point is randomly chosen from the interior of $\triangle ABC$, what is the probability that the abscissa of the point is between 3 and 5?

(A) $\frac{37}{50}$ (B) $\frac{19}{30}$ (C) $\frac{12}{91}$ (D) $\frac{2}{5}$ (E) NOTA